QUARCH: A New Quasi-Affine Reconstruction Stratum From Vague Relative Camera Orientation Knowledge

Devesh Adlakha [1,2] Adlane Habed [1] Fabio Morbidi [3] Cédric Demonceaux [2] Michel de Mathelin [1]

[1]ICube lab, CNRS, University of Strasbourg [2]ImViA lab, VIBOT ERL CNRS, University of Burgundy - Franche-Comté [3]MIS lab, University of Picardie Jules Verne

ICCV 2019 Seoul, Korea

Summary

- Accurately locating the plane at infinity ($\mathit{\Pi}_\infty$) is crucial for a successful projective-metric upgrade.
- Reliably locating $\mathit{\Pi}_\infty$ is challenging: an inherently nonlinear problem.

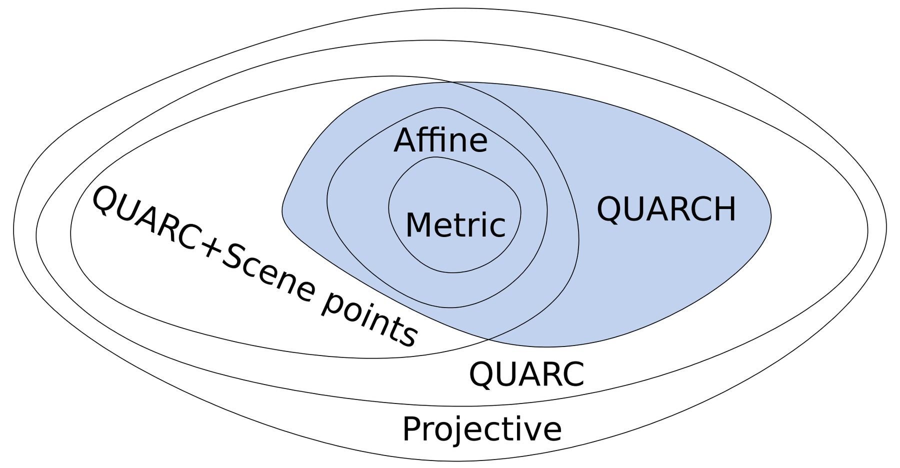
We show that for a camera pair with constant intrinsics and relative orientation angle θ_{ij}:

- the incidence relationship of $\mathit{\Pi}_\infty$ with the hodographs of the horopter depends on θ_{ij};
- based on this relationship, $\mathit{\Pi}_\infty$ satisfies either of two new sets of convex constraints depending on whether $|\theta_{ij}| \leq 120°$ or $|\theta_{ij}| \geq 120°$.

We define QUARCH, a quasi-affine reconstruction with respect to the hodographs of the horopter. It can be obtained when θ_{ij} is known to be $|\theta_{ij}| \leq 120°$ or $|\theta_{ij}| \geq 120°$ for a set of camera pairs.

We propose

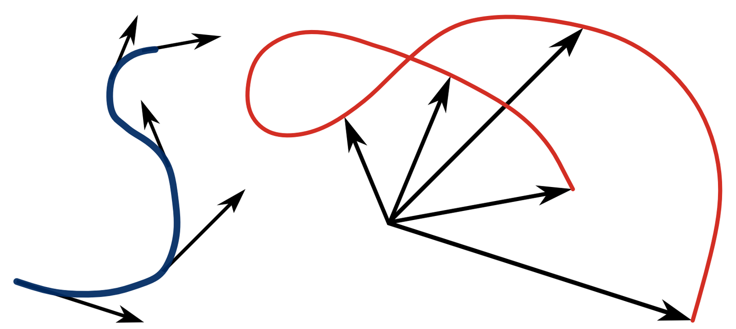
- a Semidefinite Programming (SDP) formulation to estimate a QUARCH;
- a Linear Matrix Inequality (LMI)-Constrained Levenberg-Marquardt method: to upgrade QUARCH to affine;
- a stratified camera self-calibration algorithm based on QUARCH.



Hodographs of the horopter

Horopter of a camera pair

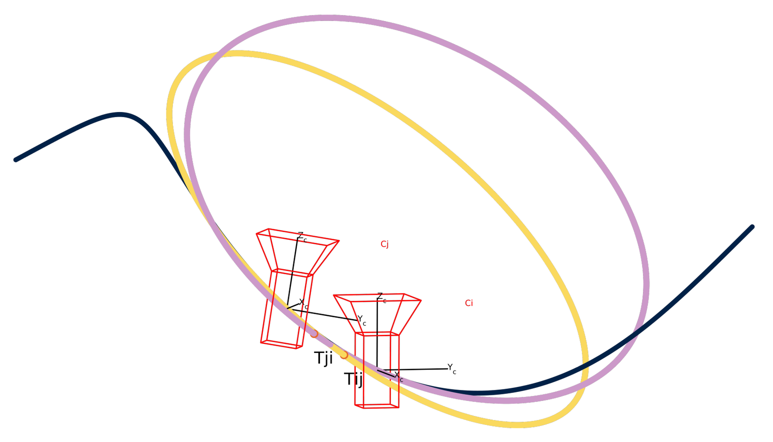
$$\mathcal{H}(s,t) = s^3\,\mathtt{C}_i - s^2 t\,\mathtt{T}_{ij} + s t^2\,\mathtt{T}_{ji} - t^3\,\mathtt{C}_j.$$

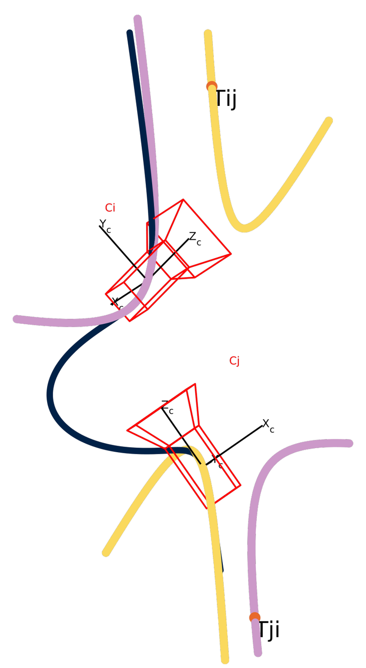


A parametric curve (blue) with its tangent vectors and its hodograph (red).

Hodographs of the horopter

$$\mathcal{H}_s(s,t) = \frac{\partial \mathcal{H}(s,t)}{\partial s}, \quad \mathcal{H}_t(s,t) = \frac{\partial \mathcal{H}(s,t)}{\partial t}$$





Horopter of a camera pair (blue) and its hodographs (yellow and purple) for $|\theta_{ij}| \leq 120°$ (left) and $|\theta_{ij}| \geq 120°$ (right).

Acknowledgements: This research was supported by the ANR SUMUM project, grant ANR-17-CE38-0004, and the ICube laboratory, CNRS.

QUARCH

- QUasi-Affine Reconstruction with respect to Camera centers and the Hodographs of horopters.
- Specialization of the QUARC stratum.

Considering sign-corrected cameras [Nistér2004] such that $\Pi_\infty^\mathsf{T}\mathtt{C}_i > 0$ and $\Pi_\infty^\mathsf{T}\mathtt{C}_j > 0$:

when $|\theta_{ij}| \leq 120°$

$\mathit{\Pi}_\infty$ intersects each hodograph in at most one real point and satisfies the LMIs:

$$\begin{bmatrix}\Pi_\infty^\mathsf{T}\mathtt{C}_i & \Pi_\infty^\mathsf{T}\mathtt{T}_{ij}\\ \Pi_\infty^\mathsf{T}\mathtt{T}_{ij} & 3\Pi_\infty^\mathsf{T}\mathtt{T}_{ji}\end{bmatrix} \succeq 0, \quad \begin{bmatrix}\Pi_\infty^\mathsf{T}\mathtt{C}_j & \Pi_\infty^\mathsf{T}\mathtt{T}_{ji}\\ \Pi_\infty^\mathsf{T}\mathtt{T}_{ji} & 3\Pi_\infty^\mathsf{T}\mathtt{T}_{ij}\end{bmatrix} \succeq 0.$$

when $|\theta_{ij}| \geq 120°$

$\mathit{\Pi}_\infty$ intersects each hodograph in at least one real point and satisfies the LMIs:

$$\begin{bmatrix}\Pi_\infty^\mathsf{T}\mathtt{C}_i & \Pi_\infty^\mathsf{T}\mathtt{T}_{ij}\\ \Pi_\infty^\mathsf{T}\mathtt{T}_{ij} & -\Pi_\infty^\mathsf{T}\mathtt{T}_{ji}\end{bmatrix} \succeq 0, \quad \begin{bmatrix}\Pi_\infty^\mathsf{T}\mathtt{C}_j & \Pi_\infty^\mathsf{T}\mathtt{T}_{ji}\\ \Pi_\infty^\mathsf{T}\mathtt{T}_{ji} & -\Pi_\infty^\mathsf{T}\mathtt{T}_{ij}\end{bmatrix} \succeq 0.$$

- A QUARCH plane satisfies these LMIs for a set of camera pairs.

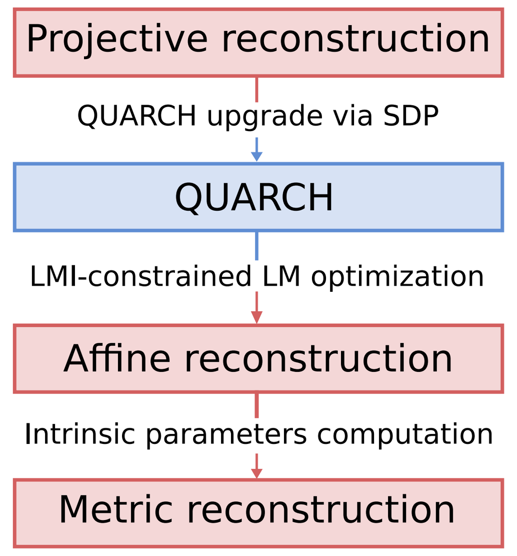
Computing a QUARCH via SDP

Assumption: $|\theta_{ij}| \leq 120°$ between consecutive views.

QUARCH SDP

$$\begin{aligned}\max_{\Pi,\,\mathtt{Z}} \quad & \log\det \mathtt{Z}\\ \text{s.t.} \quad & \mathtt{Z} \succeq 0, \quad -1 \leq (\Pi)_k \leq 1, \qquad k = 1,\ldots,4,\\ & \begin{bmatrix}\Pi^\mathsf{T}\mathtt{C}_i & \Pi^\mathsf{T}\mathtt{T}_{ij}\\ \Pi^\mathsf{T}\mathtt{T}_{ij} & 3\Pi^\mathsf{T}\mathtt{T}_{ji}\end{bmatrix} \succeq \mathtt{Z}, \quad \begin{bmatrix}\Pi^\mathsf{T}\mathtt{C}_j & \Pi^\mathsf{T}\mathtt{T}_{ji}\\ \Pi^\mathsf{T}\mathtt{T}_{ji} & 3\Pi^\mathsf{T}\mathtt{T}_{ij}\end{bmatrix} \succeq \mathtt{Z},\\ & i = 1,\ldots,n-1, \; j = i+1.\end{aligned}$$

Self-calibration



LMI-Constrained Levenberg-Marquardt method

Goal: ensure that the QUARCH LMIs are satisfied during local optimization to locate $\mathit{\Pi}_\infty$.

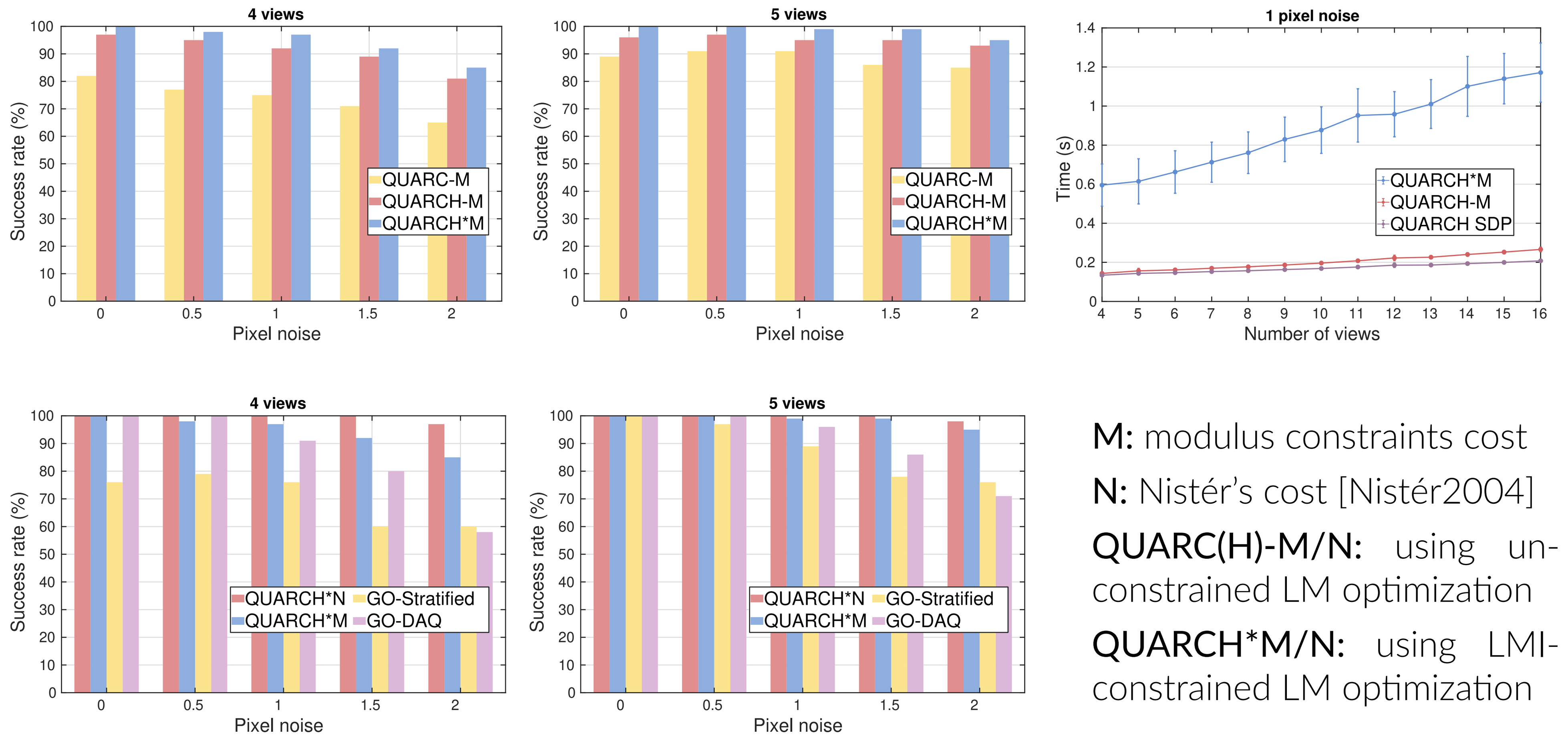
Step computation via SDP

$$\begin{aligned}\min_{d,\,\delta} \quad & \delta\\ \text{s.t.} \quad & \begin{bmatrix}J_k^\mathsf{T}J_k + \mu_k\mathtt{I}_3 & (J_k^\mathsf{T}J_k + \mu_k\mathtt{I}_3)d\\ d^\mathsf{T}(J_k^\mathsf{T}J_k + \mu_k\mathtt{I}_3) & \delta - F_k^\mathsf{T}F_k - 2F_k^\mathsf{T}J_k d\end{bmatrix} \succeq 0,\\ & x_k + d \text{ is a QUARCH plane.}\end{aligned}$$

Algorithm [Kanzow2004]

1. Choose $x_0 \in \mathcal{C}$, $\mu > 0$, and set $k = 0$.
2. If $F(x_k) = 0$, stop.
3. Choose $J_k \in \mathbb{R}^{m\times n}$, set $\mu_k = \mu\|F(x_k)\|^2$, and compute d_k as:
   $\min_d \|F(x_k) + J_k\,d\|^2 + \mu_k\|d\|^2$ s.t. $x_k + d \in \mathcal{C}$,
4. Set $x_{k+1} = x_k + d_k$, $k \leftarrow k+1$, go to step 2.

Experimental results

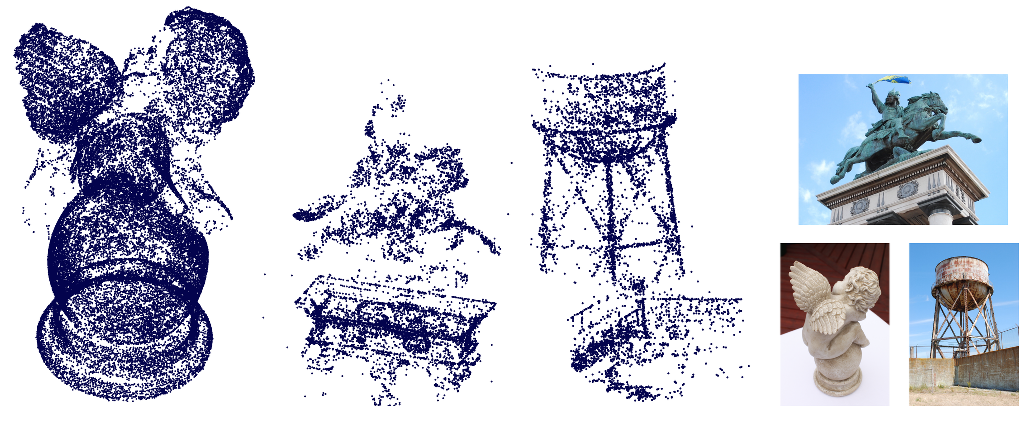


M: modulus constraints cost

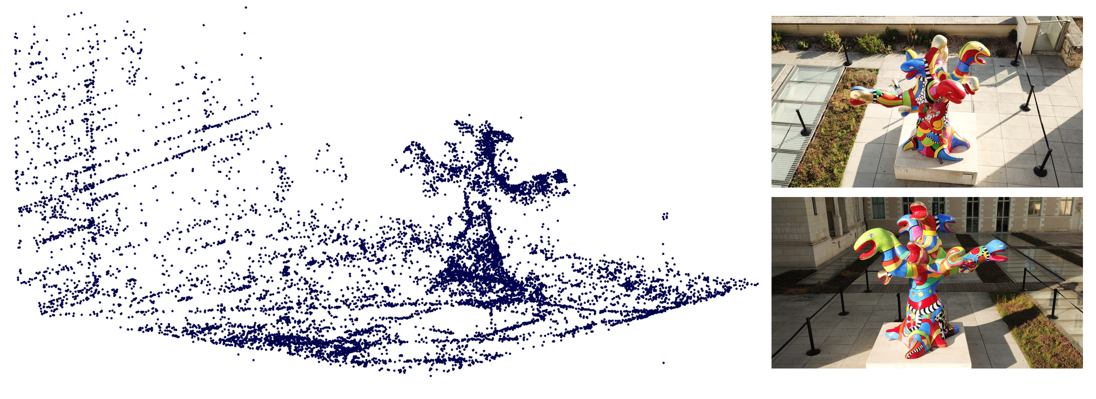
N: Nistér's cost [Nistér2004]

QUARC(H)-M/N: using unconstrained LM optimization

QUARCH*M/N: using LMI-constrained LM optimization



Metric 3D reconstructions of *Cherub*, *Vercingetorix*, and *Alcatraz water tower* obtained with QUARCH*M.



Metric 3D reconstruction of *L'Arbre aux serpents* by Niki de Saint Phalle obtained with QUARCH*M. Images courtesy Renato Saleri. (C) Musées d'Angers/ (C) 2017 Niki Charitable Art Foundation.

Herz-Jesu-P8 sequence

Method	Δf	Δuv	$\Delta\gamma$	Time (s)
QUARCH*M	0.019	0.065	1.562	1.321
QUARC-M	1.490	0.073	586.290	0.067
QUARCH*N	0.030	0.030	1.235	1.901
QUARC-N	0.028	0.030	1.211	0.091
GO-DAQ	0.024	0.028	0.267	1.853
GO-Stratified	0.845	0.653	132.849	154.735

Herz-Jesu-P25 sequence

Method	Δf	Δuv	$\Delta\gamma$	Time (s)
QUARCH*M	0.013	0.019	2.518	2.040
QUARC-M	0.013	0.019	2.545	0.446
QUARCH*N	0.021	0.028	1.404	2.084
QUARC-N	1.018	0.139	21.859	0.464
GO-DAQ	0.002	0.028	0.605	1.603
GO-Stratified	1.032	0.909	47.235	893.765

Conclusion

- Orientation-based LMI constraints on $\mathit{\Pi}_\infty$ aid in reliably obtaining a projective-metric upgrade.
- LMI-Constrained LM method could be useful for other computer vision problems.