Stratified Autocalibration of Cameras with Euclidean Image Plane

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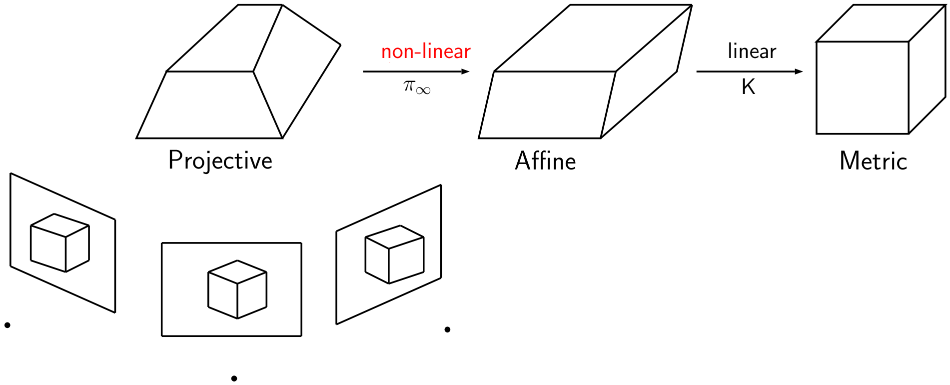
3D reconstruction from uncalibrated images



A reconstruction only up to a projective ambiguity can be obtained.

Euclidean Image Plane Assumption

A camera with square pixels *i.e.* zero skew and unit aspect ratio is said to have a Euclidean Image Plane (EIP) [Heyden and Åström, 1997].

- Most modern cameras have (very close to) square pixels
- Not fully exploited in stratified autocalibration

Assumption	Methods
Constant intrinsics	[Pollefeys and Van Gool, 1999] [Chandraker et al., 2010] [Adlakha et al., 2019]
Constant intrinsics + zero skew	[Habed et al., 2012] [Wu et al., 2013]

Contributions

Assuming a moving camera with EIP and constant intrinsic parameters,

- formulation of a new quartic polynomial in the plane at infinity, π_∞, that is obtained for each image pair $\longrightarrow$ affine reconstruction
- a stratified autocalibration method that can be used with 3 or more images

Experiments show that our method performs more reliably than existing ones.

Background

- The perspective 3×4 projection matrices (uncalibrated) are of the form,

$$\mathsf{P}_i = [\, \mathsf{H}_i \mid \mathsf{e}_i \,], \quad i = 1, 2, \ldots, n$$

  where H_i is the homography of the reference plane and e_i is the epipole.

- The inter-image homography induced by π_∞ is given as [Habed et al., 2012],

$$\mathsf{H}_{\infty ij} \;=\; \mathsf{H}_j \mathsf{H}_i^* - \mathsf{H}_j [\, \pi_\infty \,]_\times \, \mathsf{H}_i^\mathsf{T} [\, \mathsf{e}_i \,]_\times^\mathsf{T} - \mathsf{e}_j \pi_\infty^\mathsf{T} \, \mathsf{H}_i^*$$

  where H^* is the adjoint matrix of H and $[\, \pi \,]_\times$ is the skew-symmetric matrix associated with vector π. As such, $\mathsf{H}_{\infty ij}$ is linear in π_∞.

Modulus constraint

- For constant intrinsic parameters, $\mathsf{H}_{\infty ij}$ is a conjugate rotation
- Eigenvalues of $\mathsf{H}_{\infty ij}$ thus have equal moduli

Characteristic polynomial of $\mathsf{H}_{\infty ij}$,

$$\det(\mathsf{H}_{\infty 1j} - \lambda \mathsf{H}_{\infty 1i}) = -\det(\mathsf{H}_{\infty 1i})\lambda^3 + \mathrm{tr}(\mathsf{H}_{\infty ij})\lambda^2 - \mathrm{tr}(\mathsf{H}_{\infty ji})\lambda + \det(\mathsf{H}_{\infty 1j}) = 0$$

Modulus constraint: necessary condition on π_∞ for $\mathsf{H}_{\infty ij}$ to satisfy the eigenvalue property [Pollefeys and Van Gool, 1999],

$$m_{ij}(\pi_\infty) = \det(\mathsf{H}_{\infty 1i})\,\mathrm{tr}(\mathsf{H}_{\infty ji})^3 - \det(\mathsf{H}_{\infty 1j})\,\mathrm{tr}(\mathsf{H}_{\infty ij})^3 \;=\; 0 \;\text{ for all }\; i \neq j$$

m_{ij} is a quartic polynomial in π_∞.

Infinite Cayley Transform

For two cameras i and j with constant intrinsic parameters, the Infinite Cayley Transform (ICT) [Wu et al., 2013, Habed et al., 2012] is,

$$\mathsf{Q}_{\infty ij} = \lambda_j \mathsf{H}_{\infty ij} - \lambda_i \mathsf{H}_{\infty ji}$$

Properties

- $\mathsf{Q}_{\infty ij}$ is similar to a skew-symmetric matrix
- $\mathrm{tr}(\mathsf{Q}^*_{\infty ij}) > 0$
  $\longrightarrow$ combined with the modulus constraint, form necessary and sufficient conditions for $\mathsf{Q}_{\infty ij}$ to be similar to a skew-symmetric matrix [Wu et al., 2013]
- Assuming zero skew, the coordinates of the principal point (u, v) can be expressed as [Habed et al., 2012],

$$u = (\mathsf{Q}_{\infty ij})_{11} / (\mathsf{Q}_{\infty ij})_{31}, \quad v = (\mathsf{Q}_{\infty ij})_{22} / (\mathsf{Q}_{\infty ij})_{32}$$

New EIP polynomial constraint

Given a 3×3 matrix B, we define the matrix operator Φ as,

$$\Phi(\mathsf{B}) = (\mathsf{B}^* \circ \mathsf{B})_{31} + (\mathsf{B}^* \circ \mathsf{B})_{32}$$

where $\circ$ is the Hadamard (elementwise) product.

ICT property: Consider two cameras i and j with EIP and constant intrinsics,

$$\Phi(\mathsf{Q}_{\infty ij}) = 0$$

A quartic polynomial constraint on π_∞ can be derived using this property.

New EIP polynomial constraint

We observe that $\Phi(\mathsf{Q}_{\infty ij})$ expands as,

$$\Phi(\mathsf{Q}_{\infty ij}) = a_{ij}(\pi_\infty)\lambda_j^3 - b_{ij}(\pi_\infty)\lambda_i\lambda_j^2 + b_{ji}(\pi_\infty)\lambda_i^2\lambda_j - a_{ji}(\pi_\infty)\lambda_i^3 = 0$$

where the coefficients a_{ij} and b_{ij} are cubic polynomials in π_∞.

Key result: $\lambda_j^3\, a_{ij}(\pi) = \lambda_i^3\, a_{ji}(\pi)$ if the modulus constraint is satisfied. Thus,

$$p_{ij}(\pi_\infty) = -b_{ij}(\pi_\infty)\,\mathrm{tr}(\mathsf{H}_{\infty ji}) + b_{ji}(\pi_\infty)\,\mathrm{tr}(\mathsf{H}_{\infty ij}) = 0$$

p_{ij} is a new quartic polynomial in π_∞: the EIP polynomial.

Polynomial inequality constraints

- Chirality (camera centers): $c_i(\pi_\infty) > 0, \quad i = 1, \ldots, n$,

  where $c_i(\pi_\infty) = \det(\mathsf{H}_{\infty 1i})$

- ICT property: $$q_{ij}(\pi_\infty) = \mathrm{tr}(\mathsf{Q}^*_{\infty ij}) > 0, \quad \begin{array}{l} i = 1, \ldots, n-1, \\ j = i+1, \ldots, n \end{array}$$

- Principal point bounds: for an image-centered $2\overline{u} \times 2\overline{v}$ image,

$$\begin{aligned} u_{ij}(\pi_\infty) &= \overline{u}^2(\mathsf{Q}_{\infty ij})^2_{31} - (\mathsf{Q}_{\infty ij})^2_{11} \geq 0, \quad i = 1, \ldots, n-1, \\ v_{ij}(\pi_\infty) &= \overline{v}^2(\mathsf{Q}_{\infty ij})^2_{32} - (\mathsf{Q}_{\infty ij})^2_{22} \geq 0, \quad j = i+1, \ldots, n \end{aligned}$$

Estimating the plane at infinity

Homogenized polynomials: for a polynomial f of degree d in π,

$$^{h}f(\pi, \pi_4) = \pi_4^d f(\pi/\pi_4)$$

Polynomial optimization problem:

$$\begin{aligned}
\min_{\pi,\, \pi_4} \quad & \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} {}^{h}m_{ij}^2(\pi, \pi_4) + {}^{h}p_{ij}^2(\pi, \pi_4) \\
\text{s.t.} \quad & {}^{h}c_i(\pi, \pi_4) > 0, && i = 1, \dots, n, \\
& {}^{h}q_{ij}(\pi, \pi_4) > 0, && i = 1, \dots, n-1, \quad j = i+1, \dots, n, \\
& {}^{h}u_{ij}(\pi, \pi_4) \geq 0, \quad {}^{h}v_{ij}(\pi, \pi_4) \geq 0, && i = 1, \dots, n-1, \quad j = i+1, \dots, n, \\
& {}^{h}c_1(\pi, \pi_4)\, {}^{h}c_n(\pi, \pi_4) + \frac{1}{n-1} \sum_{i=1}^{n-1} {}^{h}c_i(\pi, \pi_4)\, {}^{h}c_{i+1}(\pi, \pi_4) = 1
\end{aligned}$$

Solved using Lasserre's hierarchy [Lasserre, 2008, Henrion et al., 2009].

Stratified autocalibration algorithm

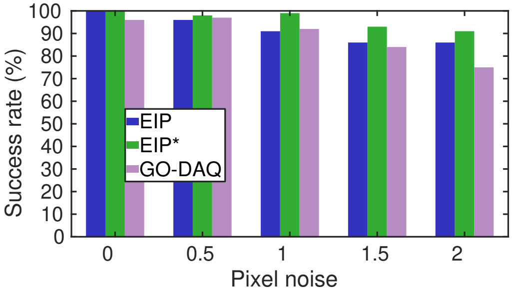
Given a projective reconstruction,

1. Estimate π_∞ by solving the polynomial optimization problem using Lasserre's hierarchy
2. Refine the estimated π_∞ using the normalized cost:

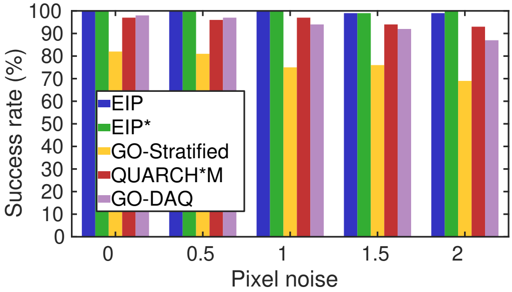
$$\pi_\infty = \arg\min_{\pi} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{m_{ij}^2(\pi) + p_{ij}^2(\pi)}{(c_i(\pi) c_j(\pi))^4}$$

3. Compute the intrinsic parameters linearly

Experimental results



(a) 3 views



(b) 4 views

EIP* our algorithm
EIP EIP* - inequality constraints

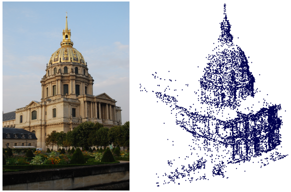
GO-Stratified [Chandraker et al., 2010]
QUARCH*M [Adlakha et al., 2019]
GO-DAQ [Chandraker et al., 2007]

Experimental results

Sequence	Method	$\Delta f(\%)$	$\Delta uv(\%)$	$\Delta\gamma$	Time (s)
fountain-P11	EIP	0.08	0.25	1.06	0.59
	GO-Stratified	0.10	0.19	1.08	302.90
	QUARCH*M	0.05	0.23	1.05	2.44
	GO-DAQ	0.36	1.26	0.01	1.49
Herz-Jesu-P8	EIP	0.55	2.84	3.98	0.57
	GO-Stratified	43.86	31.13	157.31	243.18
	QUARCH*M	0.88	3.11	2.03	1.26
	GO-DAQ	1.43	1.27	0.05	1.53
City hall Leuven	EIP	0.78	0.72	2.80	0.56
	GO-Stratified	7.09	10.10	25.85	169.21
	QUARCH*M	2.94	6.70	5.81	1.02
	GO-DAQ	9.93	7.68	9.70	1.38

Experimental results

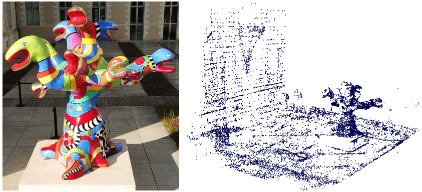
Golden Statue



Eglise du Dome

Alcatraz Water Tower

Cherub



Arbre aux Serpents[1]

[1] *L'Arbre aux Serpents de Niki de Saint Phalle* ©Musées d'Angers/ ©2017 Niki Charitable Art Foundation. Image courtesy Renato Saleri.

Summary

- Formulated a new quartic polynomial constraint on π_∞ assuming a moving camera with EIP and constant intrinsic parameters
- Our stratified autocalibration method relies on polynomial optimization and can be used with 3 or more images
- Experiments showed that our method performs more reliably than existing ones, especially for short sequences